Mathematics Standard II Preliminary Exam

Probability

- Probability is the chance of something (event) happening
- Sample space is all the possible outcomes

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes (sample space)}}$$

E.g. $P(\text{diamond}) = \frac{13}{52} = \frac{1}{4}$

Complementary events

- Complementary of an Event (E) is the event NOT including (E)

E.g. $P(4) = \frac{1}{6}$

$P(\overline{4}) = \frac{5}{6}$

Multiplication rule for counting

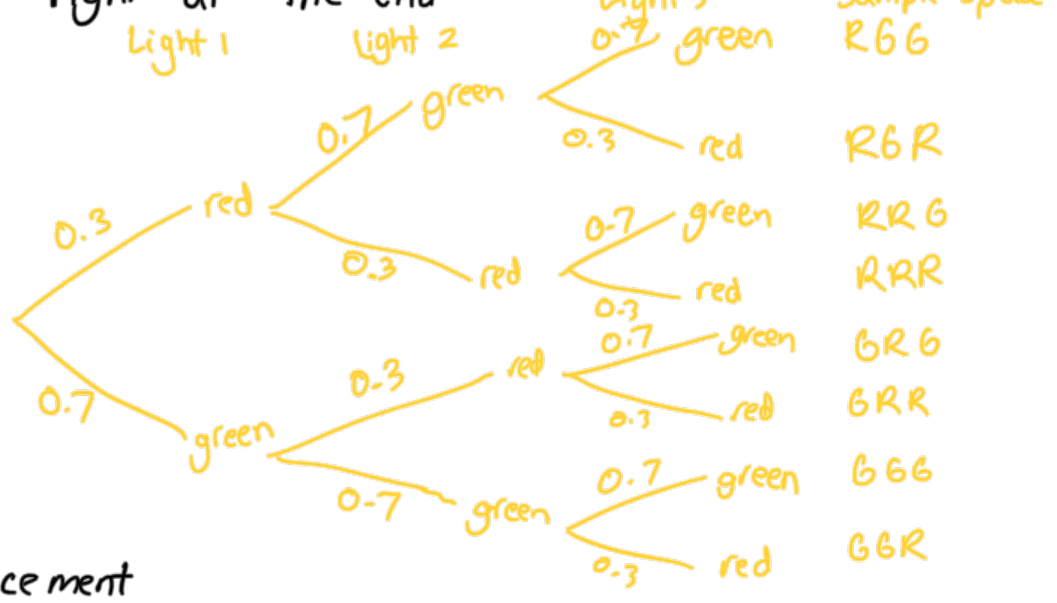
E.g. Six friends stand in a line, how many ways can they be arranged?

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways

Tree diagrams

- Lists outcomes and probabilities of a multistage experiment
- Sample space is listed on the right at the end

E.g. Die, Coin, Sample Space



- Check for replacement / non-replacement

Relative frequency, relative probability, expected probability

Relative frequency → The true probability of the event occuring

$$\text{Relative frequency} = \frac{\text{what has occured (frequency)}}{\text{total}}$$

Expected frequency → We can use relative frequency to predict

$$\text{Expected frequency} = n \times p = np$$

n: number of times the experiment is repeated

p: probability of the event

Earning and managing money

24 hrs = 1 day, 365 days = 1 year, 12 months = 1 year, 52 weeks = 1 year, fortnight = 2 weeks, 26 fortnights = 1 year

MONEY IS ALWAYS TWO DECIMAL PLACES

Overtime and special allowances

Time and a half = x1.5

Double time = x2

Annual leave and bonus pay

Holiday pay = 4 weeks pay + 17.5% of 4 weeks pay (holiday loading)

Bonus = (bonus)% x amount

Commissions, piecework and royalties

Commissions = (percentage)% x value of goods

- Advantages:
  - Higher sales increase income
  - May receive a small payment (retainer) plus the commission
- Disadvantages:
  - Income may vary each week
  - Competition for customers is typically high

Piecework = number of units of work x amount paid per unit

- Advantages
  - Incentive to work harder, income increases with more work completed
  - Often flexible hours and work place
- Disadvantages
  - No permanent employment
  - No sick leave or holiday pay
  - May not include superannuation

Royalties = percentage of the goods sold or profit received

- Advantages
  - Incentive to work hard, income increases with better product
  - Flexible hours and work place
- Disadvantages
  - Income varies according to sales
  - No superannuation, sick leave or holiday pay

Gross pay, net pay and deductions

Gross pay → total of all earnings including allowances, commissions

Gross pay → total of an employees pay including allowances, overtime pay, commissions and bonuses. The money before any deductions are made

Deductions → An amount of money subtracted from a person's wage or salary

Gross income → total amount of money earnt from all sources

Taxation

Net pay = gross pay − deductions

Taxable income = gross income − allowable tax deductions

Tax deductions → amounts allowed by the ATO to be deducted from gross income

Tax refund = tax paid − tax payable

Tax owing = tax payable − tax paid

- Medicare levy is 2% of taxable income

Equations

$a \times b = ab$

$c \times c = c^2$

$a \times a \times b = a^2b$

$a + b = a + b$

$a + a + b = 2a + b$

$2 \times a \times b = 2ab$

$3 \times a + b = 3a + b$

$ab + ab = 2ab$

$x^2 + x^2 = 2x^2$

$2ab + a^2 = 2ab + a^2$

$y^2 + x^2 + x^2 + y^2 = y^2 + x^2 + x^2 + y^2$

$-5y^2 + 8y^2 = 3y^2$

Substitution

E.g. $3a - 4bc$ $\quad a=2,\ b=5,\ c=-10$

$= 3(2) - 4(5) + (-10)$

$= 6 - 20 - 10$

$= -24$

$3a^3 \quad a=2$

$= 3(2)^3$

$= 24$

Formula applications: speed, distance, time

Stopping distance = reaction distance + breaking distance

Multi-step equations

- If: $4d = 1\frac{2}{3}$ then chance to improper fraction before solving:

$1\frac{2}{3} \times 2 \times 3 = 5$

Expansion:

E.g. $(4d+3)(2+9)$

$3(2+8v)$

Involving fractions:

$\frac{4}{x} = 12$ Multiply both sides by 'x'

$\frac{x}{3} + 3 = x + 12$ Multiply everything by the denominator

$\frac{y}{4} + \frac{y}{4} = 7$ Multiply everything by the

$\frac{3}{b+1} = 5$ Treat $(b+1)$ as a whole

3 4 LCM (lowest common multiple) and multiply both sides by it

$\frac{(+3)}{5} = \frac{2}{3}$ When two fractions are equal, cross multiply, include brackets

Changing the subject of a formula

Eg. $a^2+b^2=c^2$ Make 'a' the subject

$-b^2 \quad -b^2$

$a^2 = c^2-b^2$

$\sqrt{\ } \quad \sqrt{\ }$

$a = \sqrt{c^2-b^2}$

$A = P(1+r)^n$ Make 'r' the subject

$\div P \quad \div P$

$\frac{A}{P} = (1+r)^n$

$\sqrt[n]{\ } \quad \sqrt[n]{\ }$

$\sqrt[n]{\frac{A}{P}} = 1+r$

$-1 \quad -1$

$r = \sqrt[n]{\frac{A}{P}} - 1$

Blood Alcohol content (BAC)

$BAC_{Male} = \frac{(10N-7.5H)}{6.8M}$ $BAC_{Female} = \frac{(10N-7.5H)}{5.5M}$

N = number of standard drinks

H = hours drinking

M = Mass in (Kg) kilograms

Medication

$\text{Volume required (mL)} = \frac{\text{strength required (mg)}}{\text{stock strength (mg)}} \times \text{volume of stock (mL)}$

Practicalities of measurement

Large → Small = multiply
Small → Large = divide

Length	Weight	Capacity
mm	mg	mL
cm	g	L
m	kg	kL
km	tn	

— Errors in measurement

Precision: The smallest unit on the measuring instrument

E.g. Ruler: 1mm
Trundle Wheel: 1cm

Absolute error: $\frac{1}{2}$ of precision

If known or rounded, measured value − actual value

Relative error: Indication of how accurate the measurement is in relative to the size of the quantity of the measurement − precision ÷ actual measurement.

Express as a percentage error by multiplying by 100 − ×100

Upper bound: Measurement + $\frac{1}{2}$ precision

Lower bound: Measurement − $\frac{1}{2}$ precision

—— Scientific Notation

1. Find the first two non-zero digits
2. Place the decimal point between the digits − this is the number between 1−10
3. Count the digits between the new and old decimal point
4. Power of 10 is positive for larger numbers, negative for smaller numbers

E.g. $410\,000\,000 = 4.8 \times 10^{8}$
$0.0000004 = 4 \times 10^{-7}$
$7.689 = 7.689 \times 10^{3}$

$1.12 \times 10^{5} = 112000$ GAPS = 0s
$1.63 \times 10^{-7} = 0.000000163$

—— Significant Figures

E.g. 51.340 has 5 significant figures 5, 1, 3, 4, 0
0.00871 has 3 significant figures 8, 7, 1
56091 has 5 significant figures 5, 6, 0, 9, 1

— Significant figures in standard form

E.g. 1534000000 3 sig.fig. in standard form
$= 1.534 \times 10^{8}$
$= 1.53 \times 10^{8}$

0.000657 2 sigfig. in standard form
$= 6.57 \times 10^{-4}$
$= 6.6 \times 10^{-4}$

— Pythagoras Theorem

Pythagoras' Theorem

$h^2 = a^2 + b^2$

E.g.

$h^2 = 5^2 + 9^2$

$= 25 + 81$

$h^2 = 106$

$h = \sqrt{106}$

$= 10.295$

$= 10.30\text{cm}$

$h^2 = a^2 + b^2$

$h^2 - b^2 = a^2$

$a^2 = h^2 - b^2$

$x^2 = 12^2 - 5^2$

$= 144 - 25$

$x^2 = 119$

$x = \sqrt{119}$

$= 10.908_{mm}$

$= 10.9_{mm}$

Perimetre

Triangle	a+b+c
Quadrilateral	a+b+c+d
Square	s+s+s+s
Rectangle	2(l+b)
Circle	**2πr**

Area

Triangle	1/2×b×h
Square	s×s
Rectangle	l×b
Parallelogram	b×h
Trapezium	1/2×h×(a×b)
Rhombus	1/2×x×y
Circle	**πr²**

Area of cirdes and sectors

Big circle - small circle

↖ Annulus

↖ Sector

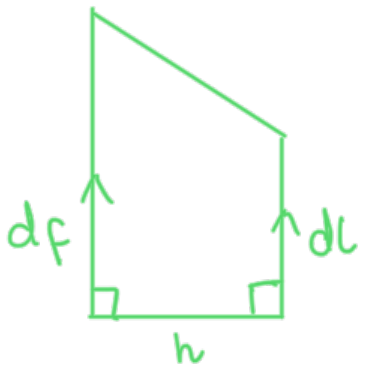
semicirde

Quadrant

Circle	πr²
Annulus	π(R²-r²)
Sector/Semicircle/Quadrant	0/360×πr²

Trapezoidal Rule

Used to estimate area



$A = \frac{h}{2} \times (df + dl)$

Eg.

$A \approx \frac{36}{2} \times (20+22)$

$\approx 756m^2$

Multiple applications:

1 application:

$A \approx \frac{36}{2} \times (22+48)$

$\approx 1260cm^2$

2 applications:

Section 1: $\frac{18}{2} \times (72+60)$

$\approx 738cm^2$

Section 2: $\frac{18}{2} \times (60+48)$

$\approx 972cm^2$

Total area: $738cm^2 + 972cm^2$

$\approx 1710cm^2$

Surface area of a cylinder and sphere

Open cylinder

closed cylinder

Sphere

open hemisphere

closed hemisphere

Sphere	$4\pi r^2$
Open hemisphere	$2\pi r^2$
Closed hemisphere	$3\pi r^2$

Surface area of a pyramid and cone

s = side length

l = slant height

Square Pyramid	$s^2+4(1/2\times s\times l)$
Cone	$\pi r^2+\pi rl$

Volume of pyramids

$V = \frac{1}{3}Ah$

A = base area

h = height

Volume of a cylinder

$V = \pi r^2 h$

∴ cross section × length/depth

Volume of a sphere

Sphere: $V = \frac{4}{3} \times \pi \times r^3$

Hemisphere: $V = \frac{4}{3} \times \pi \times r^3 \times \frac{1}{2}$

Capacity

$1L = 1000mL$

$1cm^3 = 1mL$

$1m^3 = 1000L$

Linear relationships

Graphing linear Functions/relationships/lines

Origin: (0,0)

E.g. Graph $y=2x-1$ ← substitute

x	-1	0	1
y	-3	-1	1

Testing points

- Substitute x and y into the equation

E.g. $y=-3(x)-1$

Point $(1,-4)$

$\therefore -4=-3(1)-1$

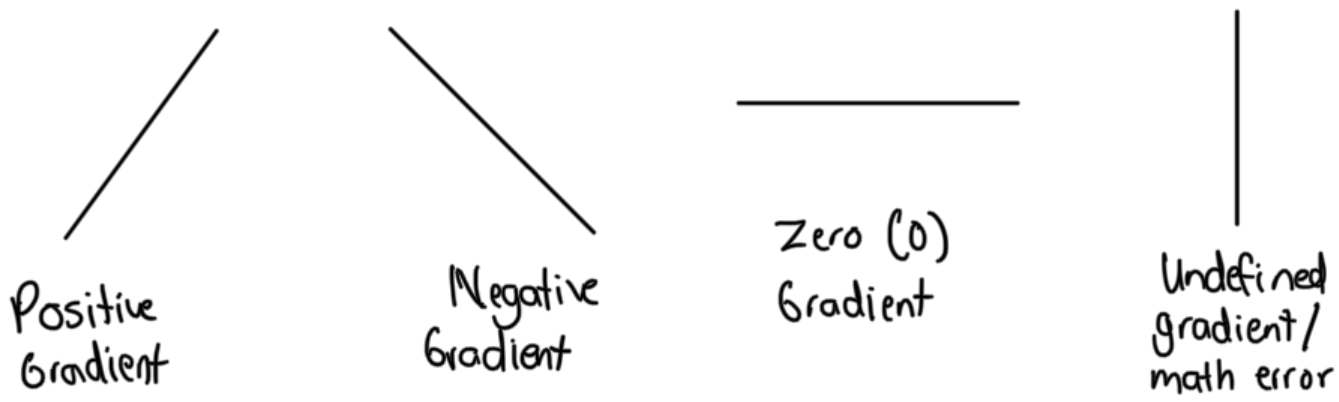
$-4=-4$

LHS = RHS

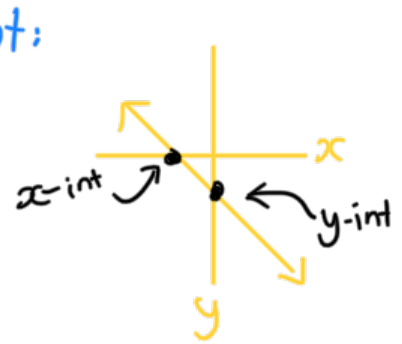
$\therefore$ The point $(1,-4)$ is on the line

Gradient

$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}}$$



Intercept:



Equation of a line

$y=mx+c$

- m = gradient
- c = y-intercept

E.g. $y=3x+1$

Gradient $=\frac{3}{1}$

y-int = 1

E.g. rearrange equations to look like $y=mx+c$:

$y=-2+5x$

$y=5x-2$

↑ ↑

m y-int

$m = \frac{3}{1}$ g...

Linear models

E.g. "A catering company charges a base amount of $100 plus a rate of $25 per guest."

C = cost
g = guest
rate of change (gradient) = 25
base amount (y-intercept) = 100
$y = mx + c$ OR table of values
$C = 25G + 100$

Directly variational / proportional

- When one variable depends on another

E.g. cost of fuel: price × litres
↑ constant

Income wages of casual staff: hour rate × hours
↑ constant

E.g. "The cost of a photographer (C, in dollars) varies directly with its speed (S, in pages per minute). A photocopier with a speed of 50 pages per minute costs $700."

- Constant variation of k: $C = kS$, $700 = k \times 50$, $\div 50$ $\div 50$, $k = 14$
- Equation connecting C and S: $c = 14 \times S$

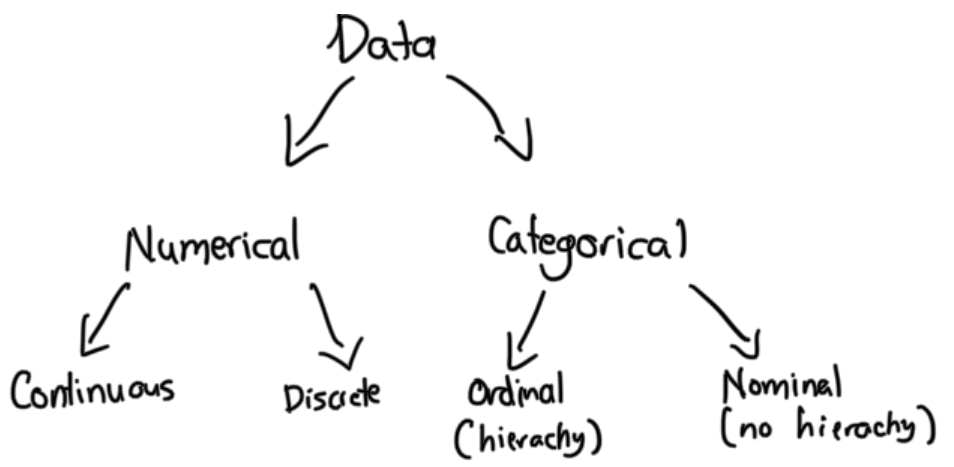
Data

Random: equal selection chance

Stratified: categories of population

Systematic: selected in certain order

self-selected: volunteers



Stem and leaf

0	1 2
1	0 0 1
2	6 8 7 9 4
3	0 2 3 4

9 6 7 8 8	0	1 4 3
4 2 2 1	1	8 9 7 1
1 4 5 6 2 1 1	2	6 1 4 2 3 4

Frequency

Frequency table

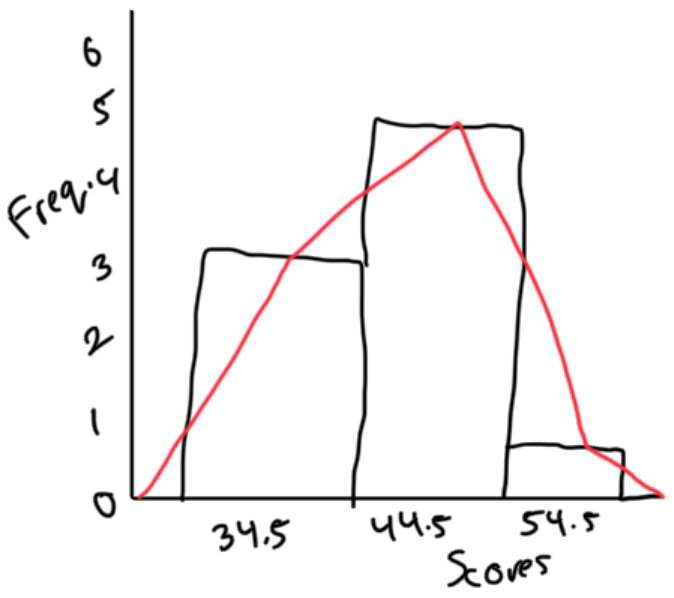
Class	Class centre	Tally	Frequency
130-139	134.5	III	3
140-149	144.5	IIII	4

Cumulative frequency table

Score	Frequency	Cumulative frequency
18	1	1
19	5	6

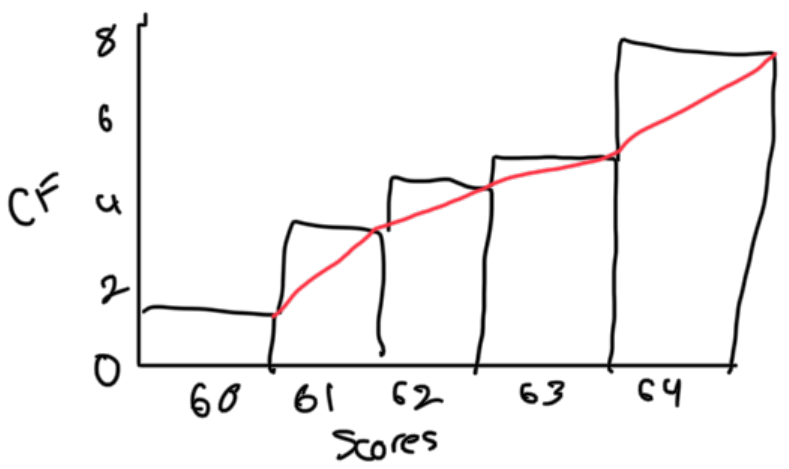
Frequency histogram and polygon

- Use class centres as grouped graph colums
- Half-colum-width space placed the vertical axis and first column



Cumulative frequency graphs

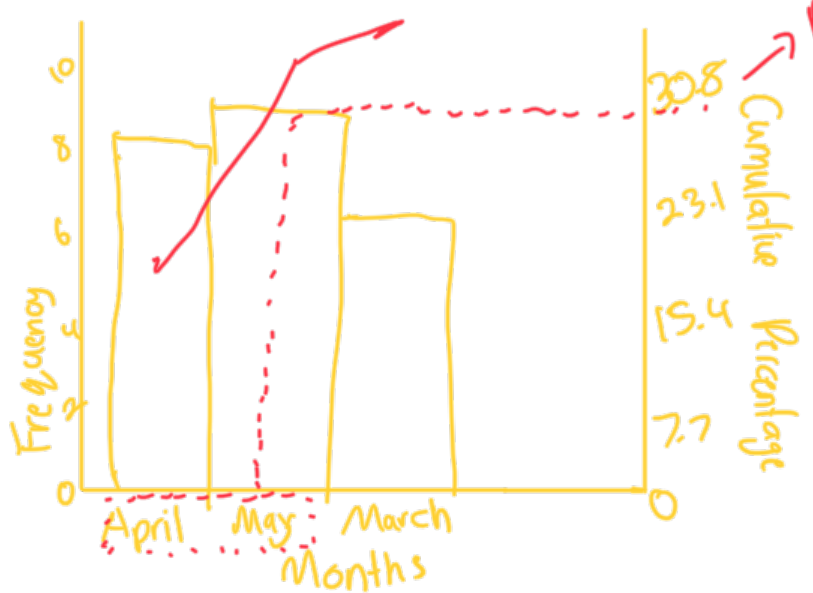
- Line from the top right of each column



Pareto charts

$$\text{Cumulative percentage} = \frac{\text{cumulative frequency}}{\text{total}} \times 100$$

Pareto principle ↴ Eg.



Pareto principle = 20% / 80%
↑ Input ↑ Output/Results

Measures of central tendancy

Mean: Average
Mode: Most frequent
Median: Middle

Mean

$$\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

Median

- List scores in ascending order **OR**
- Add scores and divide by 2

$\bar{x} = \frac{\sum fx}{\sum F}$ (for grouped data)

fx = frequency × score

F = total frequency

$\sum$ = total

If two numbers in middle, then add and divide by 2

Mode

- Most frequent number

— Measures of spread

Range: highest score − lowest score

IQR: 3rd Quartile − 1st Quartile

— Standard deviation

STAT Mode: MODE → 2 (STAT) → 1 (VAR)

Frequency Mode: SHIFT → MODE → DOWN → 3(STAT) → 1(ON)

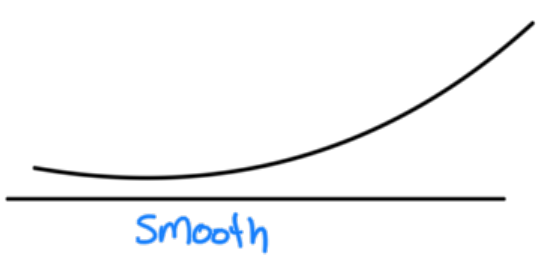
Statistics Calculations: SHIFT → 1 → 4 (VAR)

— Outliers

More than 1.5IQR range above Q3 or 1.5IQR range less than Q1

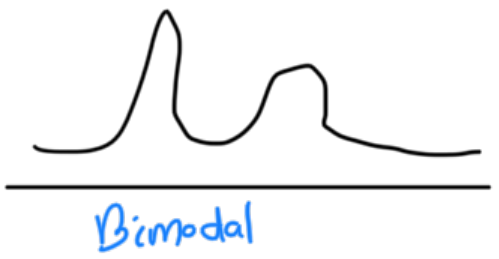
$Q1 - 1.5 \times IQR$
$Q3 + 1.5 \times IQR$
} Determine outliers

— Describing distributions

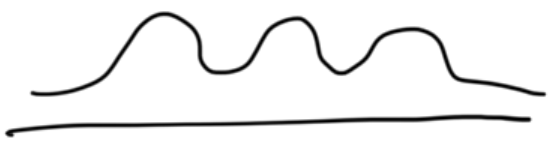


Smooth

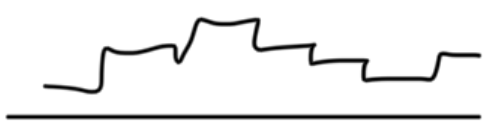
Unimodal



Bimodal



Multimodal



Jaggered

Symmetrical data

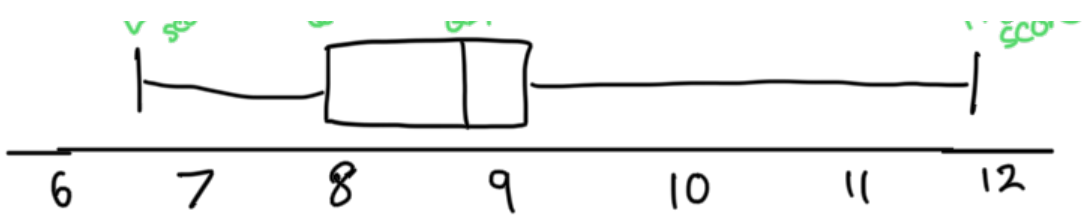
Skewed data

Positively: more data on the LEFT

Negatively: more data on the RIGHT

— Box and whisker plot

Lowest score | Q1 | Q2/Median | Highest score



Investments and depreciation

Simple interest: $I = Prn$

I = interest

P = principal

r = rate (as decimal)

n = number of years

$A = P + I$

A = amount owed / total to be paid

— Loan repayments

total to be paid ÷ number of repayments

— Simple interest graphs

E.g. "Show the simple interest earnt over 4 years if $1000 is invested at 6% p.a. Use the graph to estimate the interest earned after 8 years."

$I = 1000 \times 0.06 \times n$ ← Substitute

n = time
I = interest

n	0	1	2	3	4
I	0	60	120	180	240

∴ I = $480

— Percentage increase/decrease

Increase: $125 by 40%

$125 × 1.4

= $175

Decrease: $90 by 15%

$90 × 0.85

= $76.50

Consecutive: Increase $50 by 30% then by 20%

= ($50 × 1.3) × 1.2

= $78

— GST + VAT

GST = 0.10 or 10%

VAT = foreign tax at a percentage

Straight line depreciation

$S = V_0 - D_n$

S – salvage value / current value / book value

V_0 – purchase price / value of item if $n = 0$

D – depreciated amount per period

n – number of time periods

Purchasing a car

$$\frac{\text{New value} - \text{old value}}{\text{old value}} \times 100 \text{ (for percentage change)}$$

Fuel Consumption

$$\text{Fuel consumption} = \frac{\text{amount of fuel (L)} \times 100}{\text{distance travelled (km)}}$$